The spectrum of a cross-stream component of turbulence in a tidal stream

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The probe of a hot-film flowmeter was towed in directions approximately normal to the direction of a tidal stream. The one-dimensional spectra are found to be proportional to $k^{-\frac{5}{3}}$ in the inertial subrange.

1. Introduction

For many years, most studies of high Reynolds's number turbulence have been based upon Kolmogoroff's hypothesis (Kolmogoroff 1941) which states that in a turbulent flow of sufficiently high Reynolds's number, the small-scale motion is statistically isotropic and the form of the energy spectrum at high wavenumbers is dependent only upon the rate of dissipation of energy and the kinematic viscosity. At very high Reynolds's numbers, there is an 'inertial subrange' where the shape of the spectrum becomes independent of the viscosity.

It has been found difficult to attain a high enough Reynolds's number in laboratory flows to produce turbulence which satisfies Kolmogoroff's hypothesis. There have, however, been two experiments (Laufer 1954; Kistler & Vrebalovich 1961) in which two components of the turbulent velocity were measured in flows where the Reynolds number exceeded 10^5. In both these cases the spectrum of the downstream component showed a region where the spectral density was proportional to $k^{-\frac{5}{3}}$ where k is the wave number. That the spectrum should take this form in the inertial subrange can be predicted by a dimensional argument based on Kolmogoroff's hypothesis (Hinze 1959), but if the hypothesis is valid, the spectrum of the cross-stream component should also contain such a region. The two spectra which have been reported did not show this characteristic; they did not follow a simple power law, and the slope at high wave-numbers was less than that of the corresponding spectra of the downstream component. Since it is not certain, particularly with regard to Laufer's measurements, that the Reynolds number was high enough to satisfy Kolmogoroff's condition for the existence of an inertial subrange, it is worth examining a spectrum of this component in a geophysical flow where the Reynolds number is about 10^8.

Techniques for recording the turbulent velocity of sea water have been described by Grant, Stewart & Moilliet (1962) which will be referred to here as I. We have not yet devised a technique equivalent to the 'x-wire' which is used in air to observe the cross-stream components, but the horizontal cross-stream component can be examined by towing the probe at right angles to the stream.

This method is much simpler in principle than the use of an x-wire and, since the method of calibration is the same, the results should be directly comparable with measurements of the downstream component. It does, however, yield a component of the spectrum tensor which has not been observed before because the averaging is along a line normal to the stream.

In order to tow the probe across the stream, the ship must perform more complicated manoeuvres than are required for towing against the stream, and there is not sufficient sea room in the area where the downstream components were measured. We therefore chose a region south of Cape Mudge (see I, figure 6) for the first attempt at measurement of the cross-stream component. With a south-going tide through Discovery Passage, the current begins to turn sharply toward the east near station 7 (I, figure 6) and a mile south of the cape it is moving in an easterly direction. An extensive shoal with a depth of less than 25 m lies on the south-east corner of the cape and forces the current to maintain its easterly direction for about two miles before it turns northward and disperses.

2. Procedure

The plan was to steam into the area at about three knots on a steady course and record speed relative to the water with a current meter mounted on the towed body. The course and speed made good were to be determined by frequent optical bearings on the nearby land. Since the wind was very light, it was expected that the speed and direction of the current could be determined by simple trigonometry. This method is probably satisfactory in principle, but the turbulence was so intense and the scale of the horizontal motion was so large that the path followed by the ship was very erratic. On at least one occasion, this path included a closed loop. Under these conditions it was not possible to observe and plot positions rapidly enough with the ordinary procedures used on the bridge. The result was that the direction of the current could only be measured to within about 20° and the speed to within about 20 %.

The flowmeter and recording equipment were almost identical to that described in I, but there was an intermittent open circuit in the microphone cable with the result that the voice recorded on the tape was incomplete. Included in the lost information were the gain settings and the level of the calibration signal, so we are forced to present the spectral densities on an arbitrary scale.

3. The spectra

The record from the flowmeter contains three periods when the angle α between the towing direction and the normal to the stream was less than 40°. The experimental conditions are summarized in table 1. The water depth was about 75 m, and the probe was towed at a depth of 16 m.

The one-dimensional energy spectra obtained from the runs are given in figure 1. The scale on the ordinate is an arbitrary one, and the curves have been given an arbitrary vertical displacement relative to each other.

The curves appear to be proportional to $k^{-\frac{5}{3}}$ for about two decades in k. At the low wave-number end of the spectra, there is a marked departure from the $k^{-\frac{5}{3}}$ law and this is believed to be due to the motion of the towed body. At the other end, the three curves are very much alike and have the same shape as the spectra of the downstream component reported in I. The effects of dissipation occur at higher wave-numbers for Run 1 than for the other two,

Run	Time	Length (min)	Mean towing speed (cm/sec)	Mean current (cm/sec)	α (deg)
1	1645/4/5/61	12·6	150	230	37
2	1830/4/5/61	10·6	185	100	26
3	1855/4/5/61	15·3	150	100	10

TABLE 1. Particulars of the three runs. The mean towing speed is the speed of the towed body relative to the water and the mean current is the speed of the near-surface water relative to the bottom. α is the angle between the direction of motion of the towed body and the normal to the direction of the mean current.

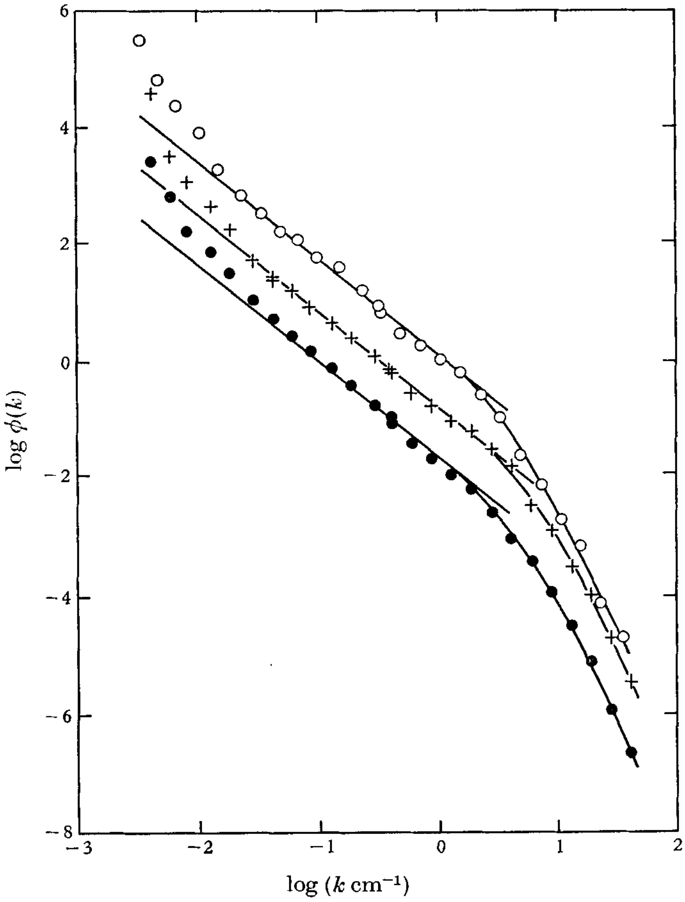

FIGURE 1. One-dimensional spectra derived from records of a cross-stream component of turbulence. From top to bottom the order of the runs is 2, 1, 3. The spectral density, $\phi(k)$, is given in arbitrary units.

suggesting that the turbulence level was higher for this run. In fact, the visible indications of turbulence on the surface were very marked at this time, and there was more than usual difficulty in steering the ship.

4. Discussion

Although the navigational accuracy leaves much to be desired, the observations leave little doubt that the spectral shape in the inertial subrange is the same for two components of the turbulence, in agreement with Kolmogoroff's theory. We hope to be able to repeat the experiment with better navigation and an absolute calibration. A direct test of local isotropy in energy, as well as spectral shape, then seems possible with regard to these two components. A statistical comparison would be necessary, however, since the two components cannot be measured simultaneously.

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